Analysis of a Ring with a Hinged Cross Section

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Theme

A SOLUTION is proposed for the small displacements of a ring assembly composed of two flanges and a web which are laced together at their intersection to form a basic channel cross section.

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In searching for a ring structure that could be folded and stored in minimum space, it has been proposed to construct a ring by lacing together three elements into a basic channel cross section. The three elements are the upper and lower flanges (see Fig. 1) and a web—all full rings of rectangular cross section. The elements are joined at the ends of the web by fine wire stitched through small holes drilled through the thickness of the flanges and web. Essentially, the lacing consists of a large number of loops joining the web to the flanges at the ends of the web.

The resulting assembly constitutes a ring whose cross section may distort in a limited sense. As a first approximation, it is assumed that the lacing 1) allows the flanges to rotate freely with respect to the web at the joints, 2) allows the flanges to slide freely in the circumferential direction with respect to the web at the joints, and 3) maintains continuity of the radial and axial components of displacement of the flanges and web at the joints.

The ring assembly is supported at three equidistant points along its periphery so as to constrain the displacement but not necessarily the rotation of the cross section about its normal. There may also exist bulkheads, located at discrete stations around the periphery, to constrain the flanges to rotate with the web. The ring assembly is assumed to be loaded by a uniform distribution of radial and transverse loads applied to the web through its centroid.

The analysis of the ring assembly is an application of the principle of virtual work taken within the framework of small displacement theory. With the components of the displacement field expressed approximately in terms of generalized displacements, the principle is used to determine the form they must take. If the generalized displacements are further expressed in Fourier series, the constraints introduced by the supports and bulkheads can be conveniently accounted for by the introduction of Lagrange multipliers.

The basic element of the ring assembly is taken to be a full ring of constant, rectangular, cross section. Arbitrary points in the cross section are located by the coordinates ξ, ζ. The coordinates ξ, ζ are centroidal and the corresponding axis are principal.

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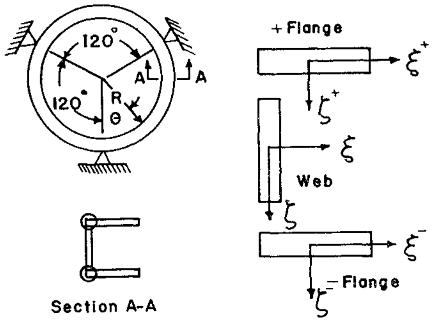

Fig. 1 Configuration (left) and coordinates (right).

The form in which the displacement components of each element are expressed is similar to that suggested by Vlasov[1] and Cheney,[2] and is given by

$$u = U(\theta) + \zeta\beta(\theta); \qquad w = W(\theta) - \xi\beta(\theta);$$

$$v = V(\theta) - \frac{\zeta}{R}\frac{dW}{d\theta} + \frac{\xi}{R}\left(V - \frac{dU}{d\theta}\right) + \frac{\phi(\xi,\zeta)}{R}\frac{d}{d\theta}(\beta + W/R)$$

The components $U(\theta)$, $W(\theta)$ can be identified as the radial, transverse displacements respectively of the centroid. The rotation of a transverse cross section about its normal is measured by $\beta(\theta)$ and $\phi(\xi, \zeta)$ is the torsion function.[3] As the torsion function is known only to within an additive constant, it is assumed that

$$\int_A \phi dA = 0$$

This assumption has the effect of defining $V(\theta)$ as the area average of the circumferential displacement.

With the displacement field in each element of the cross section described in terms of four generalized displacements U, V, W, β for each element, the total strain energy in the ring assembly and the virtual work expression can be formed. If these generalized displacements are further expressed in Fourier series, one may obtain, through the principle of virtual work, a set of algebraic equations for determining the Fourier coefficients. Care must be taken to eliminate the Fourier coefficients made dependent by the restrictions imposed on the displacement components at the joints.

The solution of the resulting equations shows that the radial and transverse deflections are uncoupled as in the case of a rigid cross section. As the solution has the same form as that for a rigid cross section, it can be concluded that the radial deflection due to a uniform in-plane load is unaffected by the lack of cross-sectional rigidity.

The solution also shows that the flange rotations (β) due to a uniform transverse load are equal. It is interesting to note that, for the case of no bulkheads, the flange rotation does not change sign between supports though the web rotation does. As expected, the effect of the bulkheads is to constrain the flange rotation to follow that of the web more closely as the number of bulkheads is increased.

Quantitative results for the transverse displacement of the web, and the flange and web rotations were obtained for a particular set of geometrical and material properties. Values were obtained corresponding to support conditions having zero deflection and either zero flange and web rotation (fixed conditions) or flange and web free to rotate but constrained to rotate together (free conditions). The effect of bulkheads was studied by introducing either one or two bulkheads symmetrically placed between each support.

The most apparent feature of the results is the strong dependency of the order of magnitude of the deflections on support condition—the transverse deflection, rotation corresponding to the free support condition are approximately ten times those of the fixed condition. Thus, in this case and for open cross sections in general, the magnitude of the deflections can be expected to depend strongly on the type of rotation support.

A comparison of the numerical results shows that the transverse deflection and web rotations of the ring assembly with bulkheads is indistinguishable from those of the ring assembly without bulkheads. It is concluded, therefore, that bulkheads have virtually no effect on the deflection and web rotations for either support condition.

A comparison is also made with the results of a similar analysis for a ring with a rigid (channel) cross section with the same dimensions. For the fixed support condition, it is shown that the transverse deflection of a ring with a rigid cross section is approximately half that of the ring assembly. The corresponding rotation is approximately 80% of the web rotation of the assembly. Thus, the ring assembly appears to be a more flexible structure for the fixed support condition.

Alternatively, for the free support condition, it is shown that there is little difference between either the rotation or the transverse deflection of either structure. Apparently, the relatively large deflections accompanying the free support condition overwhelm any increase in flexibility due to lack of cross-sectional rigidity.

In summary, the numerical results indicate that 1) bulkheads are not effective in limiting the deflections of the ring assembly; 2) the ring assembly must be prevented from rotating at the supports if the magnitudes of the deflections are to be kept as small as possible; and 3) the ring assembly may be viewed to behave in a qualitative sense as a ring with a rigid cross section, but account must be taken of cross section flexibility if quantitative results are required.

References

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[2] Cheney, J. A., "Bending and Buckling of Thin-Walled Open Section Rings," *Proceedings of the A.S.C.E, Journal of Engineering Mechanics Division*, Oct. 1965, pp. 17–44.

[3] Sokolinkoff, J. S., *Mathematical Theory of Elasticity*, 2nd ed. McGraw-Hill, New York, 1956, pp. 109–114.